Jananayak Chandrashekhar University Ballia

New

Faculty of Science

Department of Mathematics

Syllabus

(W.E.F. 2022-2023)

JANANAYAK CHANDRASHEKHAR UNIVERSITY, BALLIA

M.A./ M. Sc. MATHEMATICS
Syllabus(w.e.f. 2022-23)
Programme Name: M.Sc. Mathematics
Programme Code: PG MAT 100

With the growing role of Science and Technology in our lives, the importance of Mathematics is also increasing. A strong knowledge in mathematics has become essential for careers in the field of STEM (Science, Technology, Engineering and Mathematics). Moreover, use of Mathematics is increasing in Management, Industry, Finance, Banking and other fields.

M.Sc. Mathematics programme at Jananayak Chandrashekhar University aims at giving students a strong foundational knowledge in advanced mathematics. The programme is structured in such a manner that students can then proceed to pursue research in Mathematics as well as be skilled enough to pursue other careers requiring mathematical knowledge.

Programme Outcomes: After successfully completing this programme the student will be able to

1. Explain various concepts related to advanced mathematics.
2. Apply the knowledge gained to solve real world problems in the fields of Science, Technology, Management, Industry etc.
3. Formulate new theories and do research in mathematics.

Programme Structure:

1. The Post-Graduation programme in Mathematics of this University comprises of four semesters.
2. Each semester has four theory courses (papers), each of 5 credits and 100 marks.
3. Along with this, each student has to do one research project of 4 credits in each semester.
4. The reports of the projects carried out in 1st and 2nd semester will be compiled together and submitted in the form of Project Report/ Dissertation at the end of second semester. It will be evaluated out of 100 marks.
5. Similarly the reports of the projects carried out in 3rd and 4th semester will be compiled together and submitted in the form of Project Report/ Dissertation at the end of fourth semester. It will be evaluated out of 100 marks.
6. In 1st or 2nd semester, the student will have to study one minor elective course from a different faculty which will be of 4/5 credit.

The outline of the M.A./M.Sc. Programme is given on the following page.

M.A./M.Sc. MATHEMATICS

SEMESTER-1					
Paper	**Paper/Course Code**	**Paper/Course Name**	**Marks**	**Credits**	**Hours**
I	MAT 101	Algebra – I	100	5	75
II	MAT 102	Real Analysis	100	5	75
III	MAT 103	Basic Topology	100	5	75
IV	MAT 104	Complex Analysis	100	5	75
V	MAT 105	Project		4	

SEMESTER-2					
I	MAT 201	Algebra – II	100	5	75
II	MAT 202	Functional Analysis – I	100	5	75
III	MAT 203	Measure & integration – I	100	5	75
IV	**Elective (Optional) (Any one of the following)**				
	MAT 204	Classical Mechanics	100	5	75
	MAT 205	Special theory of Relativity	100	5	75
V	MAT 206	Project	100 (Sem-1 +Sem-2)	4	

SEMESTER-3					
I	MAT 301	Topology	100	5	75
II	MAT 302	Differential & Integral Equations	100	5	75
Paper- III& IV- **Elective (Optional) Papers (Any two of the following)**					
	MAT 303	Differential Geometry of Manifolds	100	5	75
	MAT 304	Hydrodynamics	100	5	75
	MAT 305	Operations Research	100	5	75
	MAT 306	Advanced Linear Algebra	100	5	75
V	MAT 307	Project		4	

SEMESTER-4					
I	MAT 401	Functional Analysis - II	100	5	75
II	MAT 402	Measure & Integration - II	100	5	75
Paper- III& IV- **Elective (Optional) Papers (Any two of the following)**					
	MAT 403	Complex manifolds & Contact manifolds	100	5	75
	MAT 404	Fluid Mechanics	100	5	75
	MAT 405	General Relativity and Cosmology	100	5	75
	MAT 406	Theory of optimization	100	5	75
	MAT 407	Mathematical modeling	100	5	75
V	MAT 408	Project	100 (Sem-3 + Sem-4)	4	

Note:

1. Along with the above-mentioned courses each student will have to study one minor elective course in 1st or 2nd semester from another faculty (faculty other than Science) of 4/5 credit.

SEMESTER – I

MAT 101 **ALGEBRA – I** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain various concepts related to algebra.
2. Apply results in problems arising in higher mathematics.
3. Characterize different types of groups.

UNIT I:

Action of a group G on a set S, Equivalent formulation as a homomorphism of G to T(S), Examples, Stabilizer (Isotropy) subgroups and Orbit decomposition, Class equation of an action, Its particular cases (left multiplication and conjugation), Conjugacy class equation, Core of a subgroup.Sylow's Theorem I, II and III,

UNIT II:

Subnormal and normal series, Zassenhaus's lemma (Statement only)Schreier's refinement theorem, composition series, Jordan – Holder theorem, Chain conditions, Examples, Internal and External direct products and their relationship, Indecomposability. p–groups, Examples and applications, Groups of order pq.

UNIT III:

Commutators, Solvable groups, Solvability of subgroups, factor groups and of finite p – groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

UNIT IV:

Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Unique factorization domains, Examples and counter examples, Chinese remainder theorem for rings and PID's, Polynomial rings over domains, Eisenstein's irreducibility criterion, Unique factorization in polynomial rings over UFD's.

Assignments:

1. Define group action with an example. Also define isotropy group, local isotropy group and fixed point of a group action.
2. State and prove Schreier's refinement theorem.
3. Explain solvable groups and Nilpotent groups.
4. State and prove Eisenstein's irreducibility criterion.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1.Dummit,D.S. and Foote, R. M.(2003). *Abstract Algebra.*John Wiley, N.Y.
2. Gopalakrishnan, N.S.(2015).*University Algebra* (3rd ed.).New Age Int. Pub.
3. Jacobson, N.(1984). *Basic Algebra*(Vol. 1). Hindustan Publishing Co, New Delhi.
4.Lal, R.(2002). *Algebra*(Vols. I & II). Shail Publications, Allahabad.

MAT 102 **REAL ANALYSIS** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Solve problems arising in Mathematics and Sciences.
2. Compare different types of integrations.
3. Do research and create new theories using knowledge gained in this course.

UNIT I:

Definition and existence of Riemann – Stieltjes integral, Conditions for R–S integrability. Properties of the R-S integral, R-S integrability of functions of a function Integration and differentiation, Fundamental theorem of Calculus.

Unit II:

Series of arbitrary terms. Convergence, divergence and oscillation, Absolute Convergence, Abel's and Dirichilet's tests. Multiplication of series. Rearrangements of terms of a series, Riemann's theorem and sum of series. Sequences and series of functions.

Unit III:

Pointwise and uniform convergence, Cauchy's criterion for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjies integration, Uniform convergence and differentiation, Weierstrass approximation theorem, Power series. Uniqueness theorem for power series, Abel's and Tauber's theorems.

Unit IV:

Functions of Several Variables, Linear transformations, Derivatives in an open subset of R^n Jacobian matrix and Jacobians, Chain rule and its matrix form, Interchange of order of differentiation, Derivatives of higher orders Taylor's theorem, Inverse function theorem, Implicit function theorem, Extremum problems with constraints, Lagranges multiplier method.

Assignments:

1. Explain the relation between Riemann integration and Riemann- Stieltjes integration.
2. Show that a conditionally convergent series can be made to converge to any point by proper rearrangement.
3. Discuss the difference between pointwise convergence and uniform convergence.
4. Explain Lagrange's multiplier method.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1- Rudin,W.(1976).*Principle of Mathematical Analysis*(2nd ed.).McGraw-Hill Kogakusha,International Student Edition.

2- Apostol, T. M.(1985). *MathematicalAnalysis*. Narosa Publishing House, New Delhi.

3- Lang, S. (1969). *Analysis I and II*. Addision-Wesley Pub. Co.

MAT 103 **BASIC TOPOLOGY** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain various concepts related to Metric Space and Topology.
2. Solve problems requiring knowledge of Topology.
3. Characterize different types of Topological spaces.

UNIT I:

Definition and Examples of Metric spaces, Equivalent metrics, characterization of open sets in terms of open spheres, characterization of closed sets in terms of closed spheres, Countability of a metric space, Continuity of functions, Properties of continuous functions, Homeomorphisms.

UNIT II:

Connectedness in metric spaces, Connected sets in the real line, Continuity and connectedness, Compactness, closed subset of a compact space, compact subset of a metric space, Continuity and compactness.

UNIT III:

Definition and examples of topological spaces. Closed sets. Closure. Dense sets. Neighborhoods, interior, exterior, and boundary. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology. Alternative methods of defining a topology in terms of Kuratowski closure operator and neighborhood systems.

UNIT IV:

Continuous functions and homeomorphism. First and second countable spaces. Lindelof spaces. Separable spaces. The separation axioms$T_0, T_1, T_2, T_3, T_{3\frac{1}{2}}, T_4$; their characterizations and basic properties.

Urysohn's lemma, Tietze extension theorem.

Assignments:

1. Define metric space and discrete metric space with example. Describe all open balls in discrete metric space.
2. Define Compact set in metric space with examples. Prove that compact subsets of metric space are closed.
3. Define topological space with example and its bases. Give a definition of continuous and homeomorphism function in topological spaces with suitable example.
4. State and prove the Urysohn's lemma.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Kelley, J. L.(1995). *General Topology*. Van Nostrand.
2.Joshi, K. D.(1983). *Introduction to General Topology*. Wiley Eastern.
3.Munkres, J. R.(2000).*Topology*(2nd ed.). Pearson Internationl.
4.Dugundji, J. (1966). *Topology*. Prentice-Hall of India.

MAT 104 **COMPLEX ANALYSIS** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Apply results studied in problems arising in physical sciences.
2. Relate different theorems.
3. Develop new theories and do research in higher mathematics.

UNIT I:

Analytic continuation, uniqueness of analytic continuation, Natural Boundary, complete analytic functions . Power series method ofAnalytic continuation, Schwarz's Lemma, Inverse function theorem, Schwarz's reflection principle, Reflection across analytic arcs.

UNIT II:

Residue at infinity, Cauchy's Residue theorem, Contour integration: Integral of the type$\int_{\alpha}^{2\pi+\alpha} f(cos\theta, sin\theta)d\theta, \int_{-\infty}^{\infty} f(x)dx, \int_{-\infty}^{\infty} g(x)cosmx\, dx$. Singularities on the real axis, Integrals involving branch points, Jordan's Lemma.

UNIT III:

The Riemann mapping theorem, Behavior at the boundary, Picard' theorem, Borel theorem, Infinite Products, Jensen's formula, Poission –Jenson formula, Borel Cartheodory theorem.

UNIT IV:

Entire Functions with Rational Values, The Phragmen-Lindelof and Hadamard Theorems, Meromorphic Functions. Mittag-Leffler Theorem,Weierstrass factorization theorem, Gama functions.

Assignments:

1. Discuss power series method of analytic continuation.
2. State and prove Cauchy's Residue theorem.
3. Prove the Riemann mapping theorem.
4. Write a note on Merormophic functions.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Lang, S.(1999).*Complex Analysis* (4th ed.). Springer.
2.Bak, J. and Newman, D. J.(2010).*Complex Analysis* (3rd ed.). Springer.
3. Conway, J. B.(1980).*Functions of One Complex Variable*(2nd ed.).Narosa Pub. House.

MAT 105 **PROJECT** **Credit-4**

Each student will have to complete a project in first semester. It will be of 4 credits. The evaluation of Semester I and Semester II projects will be done together at the end of second semester.

SEMESTER – II

MAT 201 **ALGEBRA – II** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Describe properties of Modules and Fields.
2. Differentiate different types of Modules.
3. Use these results in higher mathematics.

UNIT I:

Modules over a ring. Endomorphism ring of an abelian group. R-Module structure on an abelian group M as a ring homomorphism form R to End M, Submodules, Direct summands, Homomorphism, Factor modules, Correspondence theorem, Isomorphism theorems, Exact sequences, Five lemma, Products, Coproducts and their universal property, External and internal direct sums.

UNIT II:

Free modules, Homomorphism extension property, Equivalent characterization as a direct sum of copies of the underlying ring, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer's characterization.

UNIT III:

Noetherian modules and rings, Equivalent characterizations, Submodules and factors of noetherian modules, Characteristic of a field, Prime subfields, Field extension, Finite extensions, Algebraic and transcendental extensions. Factorization of polynomials in extension fields, Splitting fields and their uniqueness.

UNIT IV:

Separable field extensions, Perfect fields, Separability over fields of prime characteristic, Transitivity of separability, Automorphisms of fields, Dedekind's theorem, Fixed fields, Normal extensions, Splitting fields and normality, Normal closures, Galois extensions, extensions, Fundamental theorem of Galois theory.

Assignments:

1. Define left and right R-modules.
2. Write short note on projective and injective modules.
3. Define splitting field of a polynomial over a field. Show that the splitting field of $x^4 + 1$ over Q is $Q(\sqrt{2}, i)$.
4. Define separable and normal extensions of a field.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Dummit, D. S. and Foote, R. M.(2003). *Abstract Algebra*. John Wiley, N.Y.
2. Anderson F. W. and Fuller, K. R.(1974). *Rings and Categories of Modules*, Springer, N.Y.
3. Adamson, I. A.(1964). *An Introduction to Field Theory*. Oliver & Boyd, Edinburgh.

4. Gopalakrishnan, N. S. (2015). *University Algebra* (3rd ed.).New Age Int. Pub.
5. Hungerford, T. W.(2004). *Algebra*. Springer (India) Pvt. Ltd.
6. Lal, R. (2002).*Algebra* (Vol. 2.). Shail Publishing House, Allahabad.

MAT: 202 **FUNCTIONAL ANALYSIS - I** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Give examples of Normed linear spaces.
2. Differentiate between different types of spaces.
3. Use the results in problems arising in Functional Analysis and other branches of mathematics.

UNIT I:

Normed linear spaces, Banach spaces, their examples including $\mathbb{R}^n$, $\mathbb{C}^n$,l_p^n , l_p, C[a,b] and topological properties. Holder's and Minkowski's inqualities, Subspaces, Quotient space of a normed linear space and its completeness.

UNIT II:

Continuous linear transformations, Spaces of bounded transformations, Continuous linear functional, Hahn Banach theorems(separation and extension), strict convexity and uniqueness of Hahn Banach extension . Banach Steinhous theorem, Uniform boundedness principle.

UNIT III:

Open mapping theorem, Bounded inverse theorem, Projection,Closed graph theorem, Finite dimensional normed linear spaces, Compactness, Equivalent norms, Bolzano Weistrass property.

UNIT IV:

Duals of $\mathbb{R}^n$, $\mathbb{C}^n$,l_p^n, l_p, C[a,b], weak and weak* convergence, Embedding and reflexivity, Uniform convexity and Milman theorem.

Assignments:

1. With explanation give example of a Banach Space and a normed linear space which is not a Banach Space.
2. What are the different ways in which norm of a bounded linear transform is defined. Show that they are equivalent.
3. Show that finite dimensional normed linear spaces are always complete.
4. Find the dual of l_p space.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Simmons, G. F. (1963). *Introduction to Topology and Modern Analysis*. McGraw Hill.
2. Ponnusamy, S. (2002). *Foundation of Functional Analysis*. Narosa Publishing House.
3. Limaye, B. V.(2017).*Functional Analysis*(3rd ed.). New Age Int. Publisher.

MAT 203 **MEASURE AND INTEGRATION – I** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Compare different types of integration.
2. Select the appropriate results to use in solving problems.
3. Use the results in Mathematical research.

UNIT I:

Lebesgue outer and inner measure, Lebesgue measure on R, translation invariance of Lebesgue measure, existence of a non-measuable set, characterizations of Lebesgue measurable sets, Borel sets, Cantor-Lebesgure function.

UNIT II:

Measurable functions on a measure space and their properties, Borel measurable functions, simple functions and their integrals. Lebesgue integral on R and its properties, Riemann and Lebesgue integrals.

UNIT III:

Integral of non negative measureable function and of unbounded functions,Bounded convergence theorem. Fatou's lemma. Monotone convergence theorem, Lebesgue dominated convergence theorem.

UNITIV:

The L^p-space. Convex functions. Jensen's inequality, Holder and Minkowski inequalities, Completeness of L^p. Convergence in measure, Almost uniform convergence.

Assignments:

1. Define Cantor set and prove that the measure of Cantor set is zero.
2. State and prove Holder's Inequality and Minkowski's inequality in L^p –space.
3. State and prove Fatou'slemma.
4. State and prove Lebesgue monotone convergence theorem.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Royden, H.L. and Fitzpatrick, P.M.(2015).*Real Analysis*(4^{th} ed.). Pearson.
2. Halmos, P.R.(1994). *Measure Theory*. Springer.
3. Rana, I.K.(2005). *An Introduction to Measure and Integration* (2^{nd} ed.).Narosa Publishing House.
4. Hewit, E. and Stromberg,K.(1975). *Real and Abstract Analysis*. Springer.

Elective Papers (Optional Papers) (Any one fromMAT 204 and MAT 205)

MAT 204 **CLASSICAL MECHANICS** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Describe concepts related to mechanics.
2. Apply the results in solving problems in physical sciences.
3. Create new theories related to motion of bodies.

UNIT I:

The linear momentum and the angular momentum of a rigid body in terms of inertia constants, kinetic energy of a rigid body, equations of motion, examples on the motion of a sphere on horizontal and on inclined planes. Euler's equations of motion, motion under no forces, Eulerian angles and the geometrical equations of Euler.

UNIT II:

Generalized co-ordinates, holonomic and non-holonomic systems, configuration space, Lagrange's equations using D'Alembert's Principle for a holonomic conservative system, deduction of equation of energy when the geometrical equations do not contain time explicitly, Lagrange's multipliers case, deduction of Euler's dynamical equations from Lagrange's equations.

UNIT III:

Theory of small oscillations, Lagrange's method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints,stationary property of normal modes, Lagrange equations for impulsive motion.

UNIT IV:

Generalized momentum and the Hamiltonian for a dynamical system, Hamilton's canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, phase space and Hamilton's Variational principle, the principle of least action, canonical transformations, Hamilton-Jacobi theory, Integrals of Hamilton's equations and Poisson- Brackets, Poisson- Jacobi identity.

Assignments:

1. Define Eulerian angles and write Euler's geometrical equations.
2. Prove that if geometrical equations do not contain time t explicitly then the total energy is conserved.
3. Explain theory of small oscillations.
4. Write a note on Poisson Brackets.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Ramsey, A. S. (1985). *Dynamics: Part II.* CBS Publishers & Distributors.
2. Goldstein, H. (1969). *Classical Mechanics.* Addison-Wesley Publishing Company.
3. Rana, K. C. and Joag, P. C.(1991) *Classical Mechanics.* Tata McGraw- Hill.

MAT 205 **SPECIAL THEORY OF RELATIVITY** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain concepts related to Special Theory of Relativity.
2. Compare Newtonian Mechanics and Relativistic Mechanics.
3. Apply the results to solve problems related to relativity.

UNIT I:

Review of Newtonian Mechanics, Inertial frame, Speed of light and Galilean relativity, Michelson-Morley experiment, Lorentz-Fitzerold contraction hypothesis, relative character of space and time, postulates of special theory of relativity, Lorentz transformation equations and geometrical interpretation, Group properties of Lorentz transformations.

UNIT II:

Relativistic kinematics, composition of parallel velocities, length contraction, time dilation, transformation equations, equations for components of velocity and acceleration of a particle and contraction factor.

UNIT III:

Geometrical representation of space time, four dimensional Minkowskian space of special relativity, time-like intervals, light-like and space-like intervals, Null cone, proper time, world line of a particle, four vectors and tensors in Minkowskian space time.

UNIT IV:

Relativistic mechanics-Variations of mass with velocity, equivalence of mass energy, transformation equation for mass, momentum and energy, Energy momentum for light vector, relativistic force and transformation equation for its components, relativistic Lagrangian and Hamiltonian, relativistic equations of motion of a particle, energy momentum tensor of a continuous material distribution.

Assignments:

1. What are the postulates of Special theory of Relativity.
2. Write notes on length contraction and time dilation.
3. Establish $m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$
4. Show that the Lorentz's transformation transforms orthogonal system to non-orthogonal system.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Mollar, C.(1952). *Theory of relativity*, Clarendon press.
2.Resnick, R. (1972).*Introduction to special relativity.* Wiley Eastern Pvt. Ltd.
3.Anderson, J. L.(1967). *Principles of relativity*, Academic Press.

MAT 206 **PROJECT** **Credit-4**

Each student will have to complete a project in Second semester. It will be of 4 credits. The evaluation of Semester I and Semester II projects will be done together at the end of second semester and will consist of 100 marks.

SEMESTER – III

MAT 301 **TOPOLOGY** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Describe various concepts related to Topological Spaces.
2. Use the results in solving problems related to Topology.
3. Correlate different results.

Unit I:

Compactness. Basic properties of compactness. Compactness and finite intersection property. Sequential, countable, and B-W compactness. Local compactness. One-point compactification. Connected spaces and their basis properties. Components. Locally connected spaces. Continuity and connectedness.

Unit II:

Tychonoff product topology in terms of standard sub-base and its characterizations. Product topology and separation axioms, connected-ness, andcompactness (incl. the Tychonoff's theorem), product spaces.

Unit III:

Homotopy of paths, the fundamental group, covering spaces, fundamental group of circle, punctured plane, n-sphere, figure 8 and of surfaces.

Unit IV:

Essential and Inessential maps, equivalent conditions, Fundamental theorem of algebra, Vector fields and fixed points. Brouwer fixed point theorem for disc, Homotopy type and Jordan separation Theorem.

Assignments:

1. Explain one-point compactification of a topological space.
2. Define Tychonoff product topology in terms of standard sub-base.
3. Describe fundamental group of figure 8.
4. State and prove Jordan separation theorem.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1.Kelley, J. K. (1995). *General Topology*. Van Nostrand.

2. Joshi, K. D. (1983). *Introduction to General Topology*. Wiley Eastern.

3.Munkres, J. R.(2000). *Topology* (2nd ed.). Pearson International.

4.Dugundji, J. (1966). *Topology*. Prentice-Hall of India.

MAT 302 DIFFERENTIAL AND INTEGRAL EQUATIONS Credit-5/ Hours-75

Course Outcomes: After studying this course, students will be able to-

1. Differentiate between types of differential equations
2. Use appropriate method to solve differential equations and integral equations.
3. Create solutions of real world problems.

UNIT I:

Linear independence and Wronskians, Initial and Boundary Value Problems, Picards iterations, Lipschitz conditions, Sufficient conditions for being Lipschitzian, Examples of Lipschitzian and non-Lipschitzian functions. Picard's Theorem for local existence and uniqueness of solutions of an initial value problem of first order which is solved for the derivative, examples of problems without solutions and of equations where Picard's iterations do not converge.

UNIT II:

Pfaffian differential equations: Necessary and sufficient conditions for integrability of Total differential equation, Methods for finding solutions-by inspection, Solution of homogeneous equation, Use of auxiliary equations, solution by taking one variable as constant. Non integrable equations.

Orthogonal and Orthonormal sets of functions, Gram Schmidth orthonormalization process, Generalized Fourier Series, Sturm-Liouville problems, Examples of Boundary value problems which are not Sturm-Liouville problems.

UNIT III:

Method of separation of variables: Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations in a string of finite length and heat diffusion in a finite rod, Classification of linear integral equations, Relation between differential and integral equations.

UNIT IV:

Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Assignments:

1. Discuss the role of Wronskians in theory of differential equations.
2. Explain the method to solve Sturm-Liouville problem.
3. Find the solution of three dimensional Laplace's equation.
4. With help of an example explain the Method of successive approximation for Fredholm equation.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Sneddon, I. N.(1957).*Elements of Partial Differential Equations*. McGraw-Hill.
2. Amaranath, T.(2003).*An Elementary Course in Partial Differential Equations*. Narosa Pub.
3. Kanwal, R. P.(1997).*Linear Integral Equations*. Birkhäuser,Inc..
4. Raisinghania, M. D. (2018). *Advanced Differential Equations* (19th ed.). S. Chand.

5. Kreyszig, E. (2001). *Advanced Engineering Mathematics* (8^{th} ed.). Wiley India.

Elective Papers(Optional Papers) (Any two from MAT 303, MAT 304, MAT 305 and MAT 306)

MAT 303 DIFFERENTIAL GEOMETRY OF MANIFOLDS Credit-5/ Hours-75

Course Outcomes: After studying this course, students will be able to-

1. Describe various concepts related to Manifolds.
2. Compare different types of Manifolds.
3. Use the results to solve problems related to manifolds.

UNIT I:

Definition and examples of differentiable manifolds. Tangent spaces. Vector fields, Jacobian map Lie derivatives. Exterior algebra. Exteriorderivative, Liegroups and Lie algebras.

UNIT II:

Riemannian manifolds, Riemannian connections, Curvature tensors, Sectional curvature, Shur's theorem, Projective curvature tensor , Conformal curvature tensor, Conharmonic curvature tensor and Concircular curvature tensor.

UNIT III:

Homomorphism and isomorphism. Lie transformation groups,Principle fibre bundle, Linear fame bundle, Associated fibre bundle, Vector bundle, Tangent bundle, Induced bundle, Bundle homomorphism.

UNIT IV:

Submanifolds and Hypersurfaces, Normals, Induced connection, Gauss formulas, Weingarten formulae, Lines of curvature, Mean curvature, Generalized Gauss and Minardi-Codazzi's equations.

Assignments:

1. Define differential manifold and provide two examples of it.
2. State and prove Shur's theorem.
3. Explain tangent bundles.
4. State and prove Weingarten equations.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Mishra, R. S.(1965). *A course in tensors with applications to Riemannian Geometry*.Pothishala (Pvt.) Ltd.

2.Mishra, R. S.(1984).*Structures on a differentiable manifold and their applications*. Chandrama Prakashan, Allahabad.

3. Sinha, B. B. (1982).*An introduction to modern differentrial geometry*. Kalyani Publishes, New Delhi.

MAT 304 **HYDRODYNAMICS** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain concepts related to motion of bodies in fluids.
2. Apply this knowledge in real world problems.
3. Compare motion of different type of bodies in fluids.

UNIT I:

Equation of continuity, Boundary surfaces, streamlines, Velocity potential, Irrotational and rotational motions, Vortex lines, Euler's Equation of motion, Bernoulli's theorem, Impulsive actions.

UNIT II:

Motion in two-dimensions, Conjugate functions, Source, sink, doublets and their images, Conformal mapping, Circle Theorem.

UNIT III:

Two- dimensional irrotational motion produced by the motion of circular cylinder in an infinite mass of liquid, theorem of Blasius, Motion of Eliptic Cylinder.

UNIT IV:

Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Equation of motion of a sphere. Concentric Spheres.

Assignments:

1. Write equation of continuity in Cartesian, spherical and cylindrical coordinates.
2. Define source, sink and doublet.
3. State and prove Blasius theorem.
4. Find the motion of sphere through an infinite mass of liquid at rest at infinity.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1.Besant W. H. and Ramsey, A. S.(1988). *A Treatise on Hydrodynamics*. CBS Pub. Delhi.
2.Yuan, S. W. (1988). *Foundations of Fluid Dynamics*. Prentice-Hall of India.
3.Chorlton,F .(2009) .*Fluid Dynamics*. G. K.Publishers.

MAT 305 **OPERATONS RESEARCH** **Credit- 5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Compare different types of optimization models.
2. Apply optimization techniques to solve problems arising in management.
3. Develop new models of real world problems.

UNIT I:

Network Analysis- Minimal spanning tree, shortest path problem, Maximal Flow problem. Critical path method, Network representation, Forward pass and backward pass, slack and float, Time cost trade off. PERT, Requirements for application of PERT technique, Practical limitations in using PERT, Differences between PERT and CPM.

UNIT II:

Sequencing Problems- n jobs through 2 machines, n jobs through 3 machines, n jobs through m machines, 2 jobs through m machines. Replacement problems (Individual and group).

UNIT III:

Inventory Control: Introduction, Classification of Inventory, Economic parameter associated with inventory problems, Deterministic and Probabilistic models with and without lead time.
Queuing Theory: Structure of queuing system, Single Server Models(M/M/1), Multiple server models(M/M/C), Self-service model.

UNIT IV:

Non-Linear Programming: Introduction and definitions. Formulation of non-Linear programming problems, General non-linear programming problems. Kuhn-Tucker conditions, Lagrangian Method, Constrained optimization with equality constraints. Constrained optimization with inequality constraints. Saddle point problems Saddle points and NLPP. Wolfe's and Beale's method to solve Quadratic Programming problem.

Assignments:

1. Write a note on network representation in CPM.
2. Give example of a real world sequencing problem and solve it.
3. Discuss structure of queuing system.
4. Discuss Lagrangian method.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Taha, H. A.(2011). *Operations Research- An Introduction* (9th ed.).Pearson.
2. Rao, S. S.(1978) *Optimization- Theory and Applications*. Wiley Eastern Ltd.
3. Sharma, J.K.(2016) *Operations Research-Theory and Applications*(6th ed.). Trinity Press.

MAT 306 **ADVANCED LINEAR ALGEBRA** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain different results related to advanced Linear Algebra.
2. Use the results in solving problems related to module theory and matrices.
3. Develop new theories and do research in Linear Algebra.

UNIT I:

Algebraic and geometric multiplicities of eigenvalues, Invariant subspaces, T-conductors and T-annihilators, Minimal polynomials of linear operators and matrices, Characterization of diagonalizability in terms of multiplicities and also in terms of the minimal polynomial, Triangulability, Simultaneous triangulation and diagonalization.

UNIT II:

Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into T(M) and a free module, p-primary components, Decomposition of p-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

UNIT III:

Reduction of matrices over polynomial rings over a field,Similarity of matrices and F[x]-module structure, Projections, Invariant direct sums, Characterization of diagonalizability in terms of projections, Primary decomposition theorem.

UNIT IV:

Diagonalizable and nilpotent parts of a linear operator, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Semisimple operators, Taylor formula.

Assignments:

1. Describe algebraic and geometric multiplicities of an eigen value.
2. Write short note on torsion and torsion free module.
3. State and prove primary decomposition theorem.
4. Explain rational canonical form of a matrix.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Hofmann, K. and Kunze, R.(1971). *Linear Algebra* (2nd ed.).Pearson.
2. Dummit D. S. and Foote, R. M. (2003). *Abstract Algebra.* John Wiley & Sons.
3. Helson, H.(1994). *Linear Algebra.* Hindustan Book Agency.
4. Jacobson, N.(1984). *Basic Algebra*(Vol. I). Hindustan Publishing Co.
5. Gopalakrishnan, N. S. (2015). *University Algebra* (3rd ed.). New Age Int. Pub.
6. Hungerford, T. W.(2004). *Algebra.* Springer (India) Pvt. Ltd.
7. Musili, C.(1994). *Rings and Modules.* Narosa Publishing House.

MAT 307 PROJECT Credit-4

Each student will have to complete a project in Third semester. It will be of 4 credits. The evaluation of Semester- III and Semester-IV projects will be done together at the end of fourth semester and will consist of 100 marks.

SEMESTER- IV

MAT 401 **FUNCTIONAL ANALYSIS - II** **Credit- 5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Give examples to Inner Product Spaces.
2. Explain concepts related to Hilbert spaces.
3. Use the results in solving problems arising in Functional Analysis.

UNIT I:

Inner product spaces with example, Polarization identity, Scwartz inequality, Parallelogram law, Uniform convexity of norm induced by inner product,Orthonarmal sets, Gram-Schmidt Orthogonalisation, Hilbert spaces.

UNIT II:

Bessel's inequality, Riesz-Fisher theorem, orthonormal basis, characterization of orthonormal basis, Fourier series representation and Parsevell's relation, Separable Hilbert paces, Continuity of linear mappings, Projection theorem, Riesz- representation theorem, reflexivity of a Hilbert's space, Unique Hahn extension theorem, weak convergence and weak boundedness.

UNIT III:

Unitary operators on a Hilbert spaces, Adjoint of an operator, Self adjoint and normal operators with examples, Characterization and results pertaining to these operators, Positive operator, Shift operator, Projection on a Hilbert's space.

UNIT IV:

Finite dimensional spectral theory, Determinant and spectrum of an operator, Spectral theorem, spectral resolution.

Assignments:

1. With explanation give example of a Banach space which is not a Hilbert space.
2. Show that a Hilbert space is reflexive.
3. Show that the norm of an operator Is same as norm of its adjoint.
4. Show that every operator on a finite dimensional Hilbert space can be seen as a matrix.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Simmons,G. F.(1963).*Introduction to Topology and Modern Analysis*. McGraw Hill(India).
2. Ponnusamy, S.(2002).*Foundations of Functional Analysis*. Narosa Publishing House.
3. Limaye, B. V.(2017). *Functional Analysis*(3rd ed.). New Age Int. Publication.

MAT 402: **MEASURE AND INTEGRATION - II** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Describe different types of measures and their properties.
2. Solve problems related to measure and integration.
3. Relate various results.

UNIT I:

Semialgebras, algebras, monotone class, σ- algebras, measure and outer measures, Caratheodory extension process of extending a measure on a semi-algebra to generated σ –algebra, completion of measure space.

UNIT II:

Signed measure. Hahn and Jordan decomposition theorems. Absolutely continuous and singular measures. Radon Nikodyn theorem. Lebesgue decomposition. Riesz-Representation theorem, Extension theorem (carathedory).

UNIT III:

Product measures, Fubini's theorem, Baire sets, Baire measure, Continuous functions with compact support.

UNIT IV:

Regularity of measures on locally compact spaces. Integration of continuous functions with compact support. Riesz-Markov theorem.

Assignments:

1. Difine Measure space and give two examples.
2. Explain Caratedory extension.
3. Define product measure and give an example.
4. Prove the Riesz-Markov theorem.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1.Royden, H. L. and Fitzpatrick, P.M.(2015).*Real Analysis*(4th ed.). Pearson.
2.Halmos, P. R.(1950). *Measure Theory.* Van Nostrand.
3.Berberian, S. K.(1981). *Measure and Integration.* Wiley Eastern.
4.Taylor,A. E.,(1958). *Introduction to Functional Analysis.* John Wiley.
5. Barra, G. D.(1981). *Measure Theory and Integration.* Wiley Eastern.
6. Bartle, R. G. (1966). *The Elements of Integration.* John Wiley.
7. Rana, I. K. (2005). *An Introduction to measure and Integration* (2nd ed.). Narosa Publishing House.

Elective Papers(Optional Papers) (Any twofrom MAT 403, MAT 404, MAT 405, MAT 406 and MAT 407)

MAT 403 COMPLEX MANIFOLDS AND CONTACT MANIFOLDS **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain concepts related to complex Manifolds and contact Manifolds.
2. Differentiate between different Manifolds.
3. Apply the results to solve problems.
4. Develop new theories and do research in Manifolds.

UNIT I:

Almost complex manifolds: Elementary notions ,Nijenuis tensor, Eigen values of F, Integrability conditions, Contravariant and covariant almost analytic vectors fields, F connection.

UNIT II:

Almost Hermite manifolds: Definition, Curvature tensor, Linear connection, Kaehler manifolds: Definition, Curvature tensor, Properties of Projective, Conformal, Conharmonic and concircular curvature tensor.

UNIT III:

Almost contact manifolds: Definition, Eigen values of F, Lie derivative, Normal contact structure, Particular affine connection, Almost Sasakian manifold.

UNIT IV:

Sasakian manifolds: K- contact Riemannian manifold and its properties , Sasakian manifolds and its properties, Properties of Projective, Conformal, Conharmonic and concircularcurvature tensor in Sasakian manifolds, Cosymplectic structure.

Assignments:

1. Define complex manifolds and almost complex manifolds with examples.
2. Prove that a conharmonically flat kahler manifold is flat.
3. Write a short note on almost Sasakian manifold.
4. Define curvature tensor in Sasakian manifold.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1.Mishra, R.S.(1965).*A course in tensors with applications to Riemannian Geometry. Pothishala (P.vt.)Ltd.*
2.Mishra, R. S.(1984).*Structures on a differentiable manifold and their applications*. Chandrama Prakashan, Allahabad.
3.Sinha, B. B.(1982).*An introduction to modern differential geometry*. Kalyani Publishers, New Delhi.

MAT 408 **PROJECT** **Credit-4**

Each student will have to complete a project in fourth semester. It will be of 4 credits. The evaluation of Semester- III and Semester-IV projects will be done together at the end of fourth semester and will consist of 100 marks.

online consent
Prof. S. K. Srivastava
(Ext. Expert)

online consent
Prof. A. M. Tripathi
(Ext. Expert)

MAT 404 **FLUID MECHANICS** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Describe concepts related to motion in fluid.
2. Compare flows in different types of fluids.
3. Apply the results in solving problems related to fluids.

UNIT I:

Elementary notions of fluid motion: Body forces and surface forces, nature of stresses, Transformation of stress components, Stress invariants, Principal stresses, Nature of strains, Rates of strain components, Relation between stress and rate of strain components, General displacement of a fluid element, Newton's law of viscosity, Navier-Stokes equation (sketch of proof).

UNIT II:

Equation of motion for inviscid fluid, Energy equation, Vortex motion-Helmholtz's vorticity theorem and vorticity equation, Kelvin's circulation Theorem, Mean Potential over a spherical surface, Kelvin's Minimum kinetic energy Theorem, Acyclic irrotational motion.

UNIT III:

Wave motion in a gas. Speed of Sound. Equation of motion of a gas. Subsonic, Sonic and Supersonic flows of a gas. Isentropic gas flows.

UNIT IV:

Normal and oblique shocks. Plane Poiseuille and Couette flows between two parallel plates. Unsteady flow over a flat plate. Reynold's number.

Assignments:

1. Show that the general motion of a fluid element is made up of three parts, viz, pure translation, pure rotation and pure deformation.
2. Describe cyclic and acyclic irrotational motion.
3. Explain subsonic, sonic and supersonic flows of a gas.
4. Describe unsteady flow of viscous incompressible fluid over a flat plate.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Landau, L. D. and Lifshitz, E. M.(1987). *Fluid Mechanics*(2nd ed.). Butterworth-Heinemann.
2. Curle, N. and Davies, H.J.(1968). *Modern Fluid Dynamics*(Vol. 1).D. Van Nost. Comp London.
3.Yuan, S. W.(1967). *Foundation of Fluid Mechanics*. Prentice-Hall.
4. Ramsey,A. S.(1960). *A Treatise on Hydrodynamics*(Part). I, G. Bell and Sons Ltd.
5. Chalton, F.(1985). *A text book of fluid dynamics*. CBS Publication, New Delhi.

MAT 407 **MATHEMATICAL MODELING** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Describe Concepts related to Mathematical modeling.
2. Compare different types of mathematical models of real world problems.
3. Apply the methods to solve industrial an management problems by modeling them.

UNIT1:

Simple situations requiring mathematical modeling, techniques of mathematical modeling, Classifications, Characteristics and limitations of mathematical models, Some illustrations.

UNIT II:

Mathematical modeling through differential equations, linear growth and decay models, Non linear growth and decay models, Compartment models, Mathematical modeling in dynamics through ordinary differential equations of first order.

UNIT III:

Mathematical models through difference equations, some simple models, Basic theory of linear difference equations in economics and finance, mathematical modeling through difference eequations in polulation dynamics and genetics

UNIT IV:

Situations that can be modeled through graphs, Mathematical models in terms of Directed graphs, Mathematical models in terms of signed graphs, Mathematical models in terms of weighted graphs. Mathematical modeling through linear programming, linear programming models in forest management. Transportation and assignment models.

Assignments:

1. What are limitations of mathematical modeling.
2. Describe the population growth model.
3. Describe Gambler's ruin problems.
4. Describe mathematical models in terms of directed graphs.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Kapoor, J. N. (1988). *Mathematical Modeling.* Wiley Eastern.
2. Burghes, D. N. (1988). *Mathematical Modeling in the Social Management and Life Science.* Ellie Herwood and John Wiley.
3. Charlton, F.(1989). *Ordinary Differential and Difference Equations.* Van Nostrand.

MAT 406 **THEORY OF OPTIMIZATION** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain various optimizations methods.
2. Compare Linear and nonlinear optimization problems.
3. Apply the optimization methods to solve problems arising in industrial and management fields.

UNIT I:

Linear inequalities and theorems of the alternative, Farka's lemma, Structure of Convex Sets: Algebraic Interior and Algebraic Closure of Convex Sets, Minkowski Gauge Function, Relative Interiors of Convex Sets. Convex polyhedral, Cones, Convex cones.

UNIT II:

Convex function, Continuity of Convex Functions, Epigraph, Conjugates of convex functions, Differentiable convex functions, Concave function, Convex programming problems and its optimal solutions. Global minimum and local minimum of convex programming problem with constraints.

UNIT III:

Nonlinear Programming: The Fritz John necessary optimality conditions, Constraint Qualifications, The Karush Kuhn-Tucker (KKT) sufficient optimality conditions using convexity of constraint nonlinear programming problems. Applications of Nonlinear Programming.

UNIT IV:

Duality Theory in Nonlinear Programming, Examples of Dual Problems, Duality theorems, Generalized Convexity in Nonlinear Programming. Introduction to Semi-infinite Programming, Applications of Semi-infinite Programming.

Assignments:

1. Define a Convex set and Convex function with example. Explain Convex cone, relative interior and closure of convex set.
2. State and prove the Fritz John necessary optimality conditions for convex minimization problem using suitable constraint qualification.
3. State and prove the Karush Kuhn Tucker optimality condition using convexity of constraint nonlinear programming problem.
4. State and prove weak and strong duality theorems.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Guler,O.(2010). *Foundations of Optimization.* Springer Science+Business Media.
2. Chong, K. P. P. and Zak, S. H.(2003).*An Introduction to Optimization*(3rd ed.). Johan Welly & Sons Inc.
3. Avriel, M., Diewert, W.E., Schaible, S. and Zang, I.(2010). *Generalized Concavity*.SIAM Classics in Applied Mathematics.
4. Cambini, A. and Martein, L.(2009). *Generalized Convexity and Optimization*, Lecture Notes in Economics and Mathematical Systems Vol. 616, Springer-Verlag Berlin Heidelberg.

MAT 405 **GENERAL RELATIVITY AND COSMOLOGY** **Credit-5/ Hours-75**

Course Outcomes: After studying this course, students will be able to-

1. Explain concepts related to General Theory of Relativity.
2. Compare Newtonian Mechanics and Mechanics in curved space time.
3. Apply the results to solve problems related to general relativity.

UNIT I:

Review of special theory of relativity and the Newtonian Theory of Gravitation. Principle of equivalence and general covariance. Geodesic Principle. Newtonian approximation.

UNIT II:

Schwarzchild external solution and its isotropic form. Planetary orbits and analogues of Kepler's law in general relativity. Advance or perihelion of a planet. Bending of light rays in gravitational field. Gravitational redshift of spectral lines.

UNIT III:

Energy momentum tensor of a perfect fluid. Schwarzchild internal solution. Boundary conditions. Energy momentum tensor of an electromagnetic field. Einstein-Maxwell equations. Reissner-Nordstrom solution.

UNIT IV:

Mach's Principle. Einstein modified field equations with cosmological term. Static Cosmological models of Einstein and De-Sitter, their derivation, properties and comparison with the actual universe. Hubble's law. Cosmological principle's Wey' is postulate. Derivation of Robertson-Walke metric.

Assignments:

1. Discuss the Einstein tensor.
2. What is principle of equivalence.
3. State and prove Birkhoff's theorem.
4. Describe gravitational shift of spectral lines.

Transactional Strategies: Lectures, Group discussions, quiz, assignments.

Reference Books:

1. Weatherburn, C. E.(1950). *An Introduction To Reimanian Geometry and the tensor Calculus*. Cambridge University Press.
2. Narlikar, J. V. (1978). *General Relativity and Cosmology*. The Macmillan Company of India Ltd.
3. Prakash, S. (2014). *Relativistic Mechanics (16th ed.)*. Pragati Prakashan.
4. Roy, S.R. and Bali, R. (2008). *Theory of Relativity*. Jaipur Publishing house.