Faculty of Science

Semester Based Syllabus (w.e.f. 2018-19)

M.A./M.Sc. MATHEMATICS

Scheme of Examination

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SEMESTER – I

MAT 101 ALGEBRA – I

UNIT I: Action of a group G on a set S, Equivalent formulation as a homomorphism of G to T(S), Examples, Stabilizer (Isotropy) subgroups and Orbit decomposition, Class equation of an action, Its particular cases (left multiplication and conjugation), Conjugacy class equation, Core of a subgroup. Sylow’s Theorem I, II and III,

UNIT II: Subnormal and normal series, Zassenhaus’s lemma (Statement only) Schreier’s refinement theorem, composition series, Jordan – Holder theorem, Chain conditions, Examples, Internal and External direct products and their relationship, Indecomposability. p-groups, Examples and applications, Groups of order pq.

UNIT III: Commutators, Solvable groups, Solvability of subgroups, factor groups and of finite p – groups, Examples, Lower and upper central series, Nilpotent groups and their equivalent characterizations.

UNIT IV: Factorization theory in commutative domains, Prime and irreducible elements, G.C.D., Euclidean domains, Maximal and prime ideals, Principal ideal domains, Unique factorization domains, Examples and counter examples, Chinese remainder theorem for rings and PID’s, Polynomial rings over domains, Eisenstein’s irreducibility criterion, Unique factorization in polynomial rings over UFD’s.

Books Recommended:
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern, New Delhi, 1986.

MAT 102 REAL ANALYSIS


Unit III: Pointwise and uniform convergence, Cauchy’s criterion for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, Uniform convergence and differentiation, Weierstrass approximation theorem, Power series. Uniqueness theorem for power series, Abel’s and Tauber’s theorems.
**Unit IV:** Functions of Several Variables, Linear transformations, Derivatives in an open subset of $\mathbb{R}^n$ Jacobian matrix and Jacobians, Chain rule and its matrix form, Interchange of order of differentiation, Derivatives of higher orders Taylor’s theorem, Inverse function theorem, Implicit function theorem, Extremum problems with constraints, Lagrange's multiplier method.

**Books Recommended:**


**MAT 103 BASIC TOPOLOGY**

**UNIT I:** Definition and Examples of Metric spaces, Equivalent metrics, characterization of open sets in terms of open spheres, characterization of closed sets in terms of closed spheres, Countability of a metric space, Continuity of functions, Properties of continuous functions, Homeomorphisms.

**UNIT II:** Connectedness in metric spaces, Connected sets in the real line, Continuity and connectedness, Compactness, closed subset of a compact space, compact subset of a metric space, Continuity and compactness.


**UNIT IV:** Continuous functions and homeomorphism. First and second countable space. Lindelof spaces. Separable spaces. The separation axioms $T_0$, $T_1$, $T_2$, $T_3$, $T_{3rac{1}{2}}$, $T_4$; their characterizations and basic properties. Urysohn’s lemma, Tietze extension theorem.

**Books Recommended:**


MAT 104  COMPLEX ANALYSIS

UNIT I: Analytic continuation, uniqueness of analytic continuation, Natural Boundary, complete analytic functions, Power series method of Analytic continuation, Schwarz’s Lemma, Inverse function theorem, Schwarz’s reflection principle, Reflection across analytic arcs.

UNIT II: Residue at infinity, Cauchy’s Residue theorem, Contour integration: Integral of the type \[ \int_{\alpha} e^{-\alpha} \sin \theta \, d\theta \], Integral of the type \[ \int_{-\infty}^{+\infty} \frac{1}{1+x^2} \, dx \], Integral of the type \[ \int_{-\infty}^{+\infty} \frac{1}{1+x^2} \, dx \], Singularities on the real axis, Integrals involving branch points, Jordan’s Lemma.


Books Recommended:


MAT 105  HYDRODYNAMICS

UNIT I: Equation of continuity, Boundary surfaces, streamlines, Velocity potential, Irrotational and rotational motions, Vortex lines, Euler’s Equation of motion, Bernoulli’s theorem, Impulsive actions.

UNIT II: Motion in two-dimensions, Conjugate functions, Source, sink, doublets and their images, Conformal mapping, Circle Theorem.

UNIT III: Two-dimensional irrotational motion produced by the motion of circular cylinder in an infinite mass of liquid, theorem of Blasius, Motion of Elliptic Cylinder.

UNIT IV: Motion of a sphere through a liquid at rest at infinity. Liquid streaming past a fixed sphere, Equation of motion of a sphere. Concentric Spheres.

Books Recommended:

SEMESTER – II

MAT 201 ALGEBRA – II

UNIT I: Modules over a ring Endomorphism ring of an abelian group. R-Module structure on an abelian group M as a ring homomorphism form R to End M, Submodules, Direct summands, Homomorphism, Factor modules, Correspondence theorem, Isomorphism theorems, Exact sequences, Five lemma, Products, Coproducts and their universal property, External and internal direct sums.

UNIT II: Free modules, Homomorphism extension property, Equivalent characterization as a direct sum of copies of the underlying ring, Split exact sequences and their characterizations, Left exactness of Hom sequences and counter-examples for non-right exactness, Projective modules, Injective modules, Baer’s characterization,

UNIT III: Noetherian modules and rings, Equivalent characterizations, Submodules and factors of noetherian modules, Characteristic of a field, Prime subfields, Field extension, Finite extensions, Algebraic and transcendental extensions. Factorization of polynomials in extension fields, Splitting fields and their uniqueness.

UNIT IV: Separable field extensions, Perfect fields, Separability over fields of prime characteristic, Transitivity of separability, Automorphisms of fields, Dedekind’s theorem, Fixed fields, Normal extensions, Splitting fields and normality, Normal closures, Galois extensions, extensions, Fundamental theorem of Galois theory.

Books Recommended:


MAT: 202 FUNCTIONAL ANALYSIS I

UNIT I: Normed linear spaces, Banach spaces, their examples including $\mathbb{R}^n$, $\mathbb{C}^n$, $l^n_p$ , $l_p$, $C[a,b]$ and topological properties, Holder’s and Minkowski’s inequalities, Subspaces, Quotient spaces of normed linear spaces and its completeness.

UNIT II: Continuous linear transformations, Spaces of bounded transformations, Continuous linear functional, Hahn Banach theorems (separation and extension), strict convexity and uniqueness of Hahn Banach extension, Banach Steinhous theorem, Uniform boundedness principle.
UNIT III: Closed graph theorem, Projection, Open mapping theorem, Bounded inverse theorem, Finite dimensional normed linear spaces, Compactness, Equivalent norms, Bolzano Weistrass property.

UNIT IV: Duals of $R^n$, $C^n$, $L^p$, $C[a,b]$, weak and $weak^*$ convergence, Embedding and reflexivity, Uniform convexity and Milman theorem.

Books Recommended:


MAT 203 MEASURE AND INTEGRATION – I

UNIT I: Lebesgue outer and inner measure, Lebesgue measure on $R$, translation invariance of Lebesgue measure, existence of a non-measurable set, characterizations of Lebesgue measurable sets, Borel sets, Cantor-Lebesgue function.

UNIT II: Measurable functions on a measure space and their properties, Borel measurable functions, simple functions and their integrals, Lebesgue integral on $R$ and its properties, Riemann and Lebesgue integrals.

UNIT III: Integral of non negative measurable function and of unbounded functions, Bounded convergence theorem, Fatou’s lemma, Monotone convergence theorem, Lebesgue dominated convergence theorem.


Books Recommended:


MAT 204 CLASSICAL MECHANICS

UNIT I: The linear momentum and the angular momentum of a rigid body in terms of inertia constants, kinetic energy of a rigid body, equations of motion, examples on the motion of a sphere on horizontal and on inclined planes. Euler’s equations of motion, motion under no forces, Eulerian angles and the geometrical equations of Euler.
UNIT II: Generalized co-ordinates, geometrical equations, holonomic and non-holonomic systems, configuration space, Lagrange’s equations using D’Alembet’s Principle for a holonomic conservative system, deduction of equation of energy when the geometrical equations do not contain time explicitly, Lagrange’s multipliers case, deduction of Euler’s dynamical equations from Lagrange’s equations.

UNIT III: Theory of small oscillations, Lagrange’s method, normal (principal) co-ordinates and the normal modes of oscillation, small oscillations under holonomic constraints, stationary property of normal modes, Lagrange equations for impulsive motion.

UNIT IV: Generalized momentum and the Hamiltonian for a dynamical system, Hamilton’s canonical equations of motion, Hamiltonian as a sum of kinetic and potential energies, phase space and Hamilton’s Variational principle, the principle of least action, canonical transformations, Hamilton-Jacobi theory, Integrals of Hamilton’s equations and Poisson-Brackets, Poisson-Jacobi identity.

Books Recommended:


MAT 205 MATHEMATICAL METHODS

UNIT I: (Fourier Series)

Periodic functions, Trigonometric series, Fourier series, Euler formulas, Functions having arbitrary periods, Even and odd functions, Half-range expansions, Determination of Fourier coefficients without integration, Approximation by trigonometric polynomials, Square error.

UNIT II: (Boundary-value problems and Transforms) Orthogonal and Orthonormal sets of functions, Generalized Fourier series, Sturm-Liouville problems, Examples of Boundary-value problems which are not Sturm-Liouville problems, Definition, Existence and Linearity of Laplace Transform.

UNIT III: (Fourier Transform) Fourier Integrals, Fourier Cosine and Sine Integrals, Inverse Fourier Transform, Fourier Cosine and Sine Transform, Complex form of the Fourier Transform, Linearity of the Fourier Transform.

UNIT IV: Calculus of Variations: Functionals and extremals, Variation and its properties, Euler equations, Cases of several dependent and independent variables, Functionals dependent on higher derivatives, Parametric forms, Simple applications.
Books Recommended:


SEMESTER – III

MAT 301 TOPOLOGY


Unit II: Tychonoff product topology in terms of standard sub-base and its characterizations. Product topology and separation axioms, connected-ness, and compactness (incl. the Tychonoff’s theorem), product spaces.

Unit III: Homotopy of paths, the fundamental group, covering spaces, fundamental group of circle, punctured plane, n-sphere, figure 8 and of surfaces.

Unit IV: Essential and Inessential maps, equivalent conditions, Fundamental theorem of algebra, Vector fields and fixed points, Brouwer fixed point theorem for disc, Homotopy type and Jordan separation Theorem.

Books Recommended:

MAT 302 ADVANCED LINEAR ALGEBRA

UNIT I: Algebraic and geometric multiplicities of eigenvalues, Invariant subspaces, T-conductors and T-annihilators, Minimal polynomials of linear operators and matrices, Characterization of diagonalizability in terms of multiplicities and also in terms of the minimal polynomial, Triangulability, Simultaneous triangulation and diagonalization.

UNIT II: Submodules of finitely generated free modules over a PID, Torsion submodule, Torsion and torsion-free modules, Direct decomposition into T(M) and a free module, p-primary components, Decomposition of p-primary finitely generated torsion modules, Elementary divisors and their uniqueness, Decomposition into invariant factors and uniqueness, Direct sum decomposition of finite abelian groups into cyclic groups and their enumeration.

UNIT III: Reduction of matrices over polynomial rings over a field, Similarity of matrices and F[x]-module structure, Projections, Invariant direct sums, Characterization of diagonalizability in terms of projections, Primary decomposition theorem.

UNIT IV: Diagonalizable and nilpotent parts of a linear operator, Rational canonical form of matrices, Elementary Jordan matrices, Reduction to Jordan canonical form, Semisimple operators, Taylor formula.

Books Recommended:

5. N.S. Gopalakrishnan, University Algebra, Wiley Easten, New Delhi, 1986.

MAT 303 DIFFERENTIAL AND INTEGRAL EQUATIONS

UNIT I: Solution of differential equations in ascending and descending power series, Frobenius method, Hypergeometric Differential equations, Pochhammer symbol, Hypergeometric Function, Solution of Gauss’s Hypergeometric differential equation, Differentiation of Hypergeometric functions, Solution of Legendre’s and Bessel’s differential equation, Legendre’s and Bessel’s function.

UNIT II: Generating function for $P_n(x)$, Laplace definite integrals for $P_n(x)$, Orthogonal properties of Legendre’s polynomials, Recurrence formulae, Beltrami result, Christoffel’s expansion and summation formulae, Rodrigue’s formula for $P_n(x)$, Generating function for $J_n(x)$ Recurrence formulae for $J_n(x)$, Orthogonality of Bessel’s function.
UNIT III: Method of separation of variables: Laplace, Diffusion and Wave equations in Cartesian, cylindrical and spherical polar coordinates, Boundary value problems for transverse vibrations in a string of finite length and heat diffusion in a finite rod, Classification of linear integral equations, Relation between differential and integral equations.

UNIT IV: Fredholm equations of second kind with separable kernels, Fredholm alternative theorem, Eigen values and eigen functions, Method of successive approximation for Fredholm and Volterra equations, Resolvent kernel.

Books Recommended:
4. V.S. Verma, Series solution and special functions, Neelkamal Publication

MAT 304 DIFFERENTIAL GEOMETRY OF MANIFOLDS


UNIT II: Riemannian manifolds, Riemannian connections, Curvature tensors, Sectional curvature, Shur’s theorem, Projective curvature tensor, Conformal curvature tensor, Conharmonic curvature tensor and Concircular curvature tensor.


UNIT IV: Submanifolds and Hypersurfaces, Normals, Induced connection, Gauss formulas, Weingarten formulae, Lines of curvature, Mean curvature, Generalized Gauss and Minardi-Codazzi’s equations.

Books Recommended:
MAT 305   OPERATIONS RESEARCH


UNIT II: Transformation and Assignment problems, Integer programming, Dynamic Programming.


UNIT IV: Non linear programming-one and multi variable unconstrained optimization, Conditions for constrained optimization, Separable programming, Convex programming, Non-convex programming, Linear Goal programming.

Books Recommended:


SEMESTER- IV

MAT 401  
FUNCTIONAL ANALYSIS II

UNIT I: Inner product spaces with example, Polarization identity, Schwartz inequality, Parallelogram law, Uniform convexity of norm induced by inner product, Orthonormal sets, Gram-Schmidt Orthogonalisation.


UNIT III: Unitary operators on a Hilbert spaces, Adjoint of an operator, Self adjoint and normal operators with examples, Characterization and results pertaining to these operators, Positive operator, Shift operator, Projection on a Hilbert’s space.

UNIT IV: Finite dimensional spectral theory, Determinant and spectrum of an operator, Spectral theorem, spectral resolution.

Books Recommended:


MAT 402:  
MEASURE AND INTEGRATION II

UNIT I: Semi algebras, algebras, monotone class, σ-algebras, measure and outer measures, Caratheodory extension process of extending a measure on a semi-algebra to generated σ-algebra, completion of measure space.


UNIT III: Product measures, Fubini’s theorem, Baire sets, Baire measure, Continuous functions with compact support.

UNIT IV: Regularity of measures on locally compact spaces. Integration of continuous functions with compact support. Reisz-Markoff theorem.
Books Recommended:


MAT 403      COMPLEX MANIFOLDS AND CONTACT MANIFOLDS

UNIT I: Almost complex manifolds: Elementary notions ,Nijenius tensor, Eigen values of F, Integrability conditions, Contravariant and covariant almost analytic vectors fields, F connection.

UNIT II: Almost Hermite manifolds: Definition, Curvature tensor, Linear connection

Kaehler manifolds: Definition, Curvature tensor, Properties of Projective, Conformal, Conharmonic and concircular curvature tensor.

UNIT III: Almost contact manifolds: Definition, Eigen values of F, Lie derivative, Normal contact structure, Particular affine connection, Almost Sasakian manifold.


Books Recommended:

UNIT I: Elementary notions of fluid motion: Body forces and surface, Forces nature of stresses, Transformation of stress components, Stress invariants, Principal stresses, Nature of strains, Rates of strain components, Relation between stress and rate of strain components, General displacement of a fluid element, Newton’s law of viscosity, Navier-Stokes equation (sketch of proof).

UNIT II: Equation of motion for inviscid fluid, Energy equation, Vortex motion-Helmholtz’s vorticity theorem and vorticity equation, Kelvin’s circulation Theorem, Mean Potential over a spherical surface, Kelvin’s Minimum kinetic energy Theorem, Acyclic irrotational motion.


UNIT IV: Normal and oblique shocks. Plane Poiseuille and Couette flows between two parallel plates. Unsteady flow over a flat plate. Reynold’s number.

Books Recommended: